Afforestation as a real option with joint production of environmental services

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ABSTRACT

Real option applications in conservation have showed that with irreversibility and uncertainty about the value of preservation decisions may change. More specifically, returns must be high enough to also pay out the value of waiting if conversion into more intensive land uses is to become optimal. However, many environmental policies today focus on nature restoration, where conversion has previously taken place. In this study, we therefore reverse the problem and ask when to afforest productive agricultural land, when we face uncertainty about the value of ecosystem services delivered by afforestation. Furthermore, projects such as afforestation are often associated with joint production of forest products and environmental goods, like biodiversity, hunting, groundwater production, carbon storage, recreation etc. Thus, we extend state-of-the-art models to handle two additive ecosystem services, which both are uncertain and may be correlated. The joint production aspect increases the value of conversion, the stopping value, and hence the incentives to afforest. Increasing uncertainty decreases this incentive, as expected. However, contrary to the existing literature evaluating exclusive options, less than perfect correlation between the values of future ecosystem services decreases the value of the real option and increases the set of states, where afforestation is the preferred decision. This causes afforestation to be more attractive for a wider set of states of the world than otherwise and has implications where joint production is feasible. We discuss these findings and the potential application of this analysis for handling real options with joint production in other research domains.

1. Introduction

Around the world, many governments and non-governmental organisations have been engaged in afforesting agricultural and wasteland or reforesting harvested forest stands. There are many reasons for implementing policies, which promote this land use conversion. A range of socioeconomic benefits are expected from afforestation, for example afforestation may result in higher recreational benefits, reduce erosion and desertification problems, protect groundwater, create a supply of fuel wood and timber and contribute to carbon storing and climate change mitigation. The global investments are significant. More than 14 million hectares are annually afforested and reforested (FAO, 2015). To initiate such investment, decision makers at any level from governments to landowners, need to evaluate whether the afforestation is a financially more attractive investment than other land use alternatives. In particular when the investment is subject to great uncertainty about future costs and benefits, irreversibility in the sense of not being able to fully recover investments, and the benefits may be multiple and jointly produced (Nelson et al., 2009). This study discusses and analyses this problem of deciding for what states of nature to invest in afforestation when the values of ecosystem services from forests are stochastic and jointly produced. We analysed this in the context of real option theory and add to the research field by applying a new framework of additive real options. Our focus is in a sense the reverse of earlier studies, focused on when to intensify land use and halt conservation (e.g. Conrad, 1997; Kassar and Lasserre, 2004), as we here focus on reducing land use intensity, switching land use from agriculture to forest.

Even newly established afforestation areas quickly provide a wide range of ecosystem services such as recreation or increased quality of drinking water production (Vesterdal et al., 2002; Zandersen et al., 2007), and increasing the forest area on a property may increase hunting values (Meilby et al., 2006; Lundhede et al., 2015). Other services from afforestation may only happen in the long term, e.g. reaching a suitable habitat for important biodiversity (Verheyen, 2007) or significant returns from timber harvest. In valuing
such outputs/services of multifunctional forestry, it is important to account for the joint production of services. The joint production properties may fall into categories of fixed, complementarity, independence, and competitive relationships between the services (Randall, 2002). Ignoring these properties, any attempt to value the services provided by forest reserves may fail and inappropriate information may flow into relevant decision processes. The Sustainable Development Goals and the 2030 agenda highlight sustainable use of terrestrial ecosystems and management of forests. The EU Commission has acceded to major international declarations, recommendations, treaties or conventions of immediate relevance to sustainable forest and nature management, e.g. the EU Forest Action Plan and the Strategy of Lisbon’s environmental pillar, the Convention on Biological Diversity, and has strongly supported the promotion of forest management through its programmes. The EU and EU member countries annually spend billions of euros on contributing to development of sustainable forest and nature management to halt the decline of biodiversity and promote the provision of ecosystem services (e.g. see the EU Natura 2000). The Millennium Ecosystem Assessment of nature’s contributions to human well-being (Braat and de Groot, 2012; Carpenter et al., 2006; Pascual et al., 2017; TEEB, 2010) as well as the climate change debate and discussion on the potential roles of forests have increased the research community interest in several issues. Topics include the role of spatial correlation, spatial connection/dis-connection, and potential joint production of provisional services (e.g. forest and agricultural production, drinking water), regulatory services (e.g. carbon, nutrient cycling, biodiversity) or cultural services (e.g. hunting, recreation) and potential for win-win solutions and spin-offs of conservation and restoration projects (Bateman et al., 2015; Carpenter et al., 2006; Nelson et al., 2009; Venter et al., 2009).

From society’s point of view as well as the private land owner’s perspective, it is therefore an issue that the value of these possible effects and correlations are not well known. Nor are the quantitative measures of their provision. Consequently, the values associated with afforestation areas are uncertain. The expected value of e.g. future biodiversity may increase as society grows richer (Jacobsen and Hanley, 2009), as may marketed recreational services like hunting (Lundhede et al., 2015; Meilby et al., 2006). The supply of resources may become increasingly scarce (Schorer et al., 2005). On the other hand, the perceived and expected future value of a specific afforestation areas’ value may also decline. An example is if e.g. biophysical limits to growth cause demand from non-consumptive benefits to decrease relative to consumptive goods. Or if changes in surrounding land use decrease the quality and value of ecosystem services provided from the afforestation area. At the same time, the afforestation investment may be seen as irreversible. Converting back to agriculture likely implies a larger cost of removing tree stumps and re-establishing the agricultural field, the more time has passed. As time passes the trees grow valuable which reduce net cost of conversion back to agriculture. However, in many instances legislation may prohibit such conversion. Thus, setting aside agricultural land for afforestation may be seen as a decision problem of when to exercise the option of not only one, but the sum of several jointly produced ecosystem services. The value of which is governed by possibly correlated stochastic processes. Here we address this problem applying two additive real options as an example, recreational hunting and forest products.

The use of real option value models and the timing of (dis-)investment decisions have been investigated in resource economics, building on e.g. McDonald and Siegel (1986) and picking up speed with Dixit and Pindyck (1994). Specifically concerning the issues of different option values and preservation, Arrow and Fisher (1974) and Henry (1974) pointed out the (quasi-) option value related to continued preservation (as opposed to development) of a resource when benefits from development are uncertain. As already mentioned, the problem we analysed here is the reverse one of earlier studies focused on when to intensify land use and halt conservation (e.g. Conrad, 1997; Kassar and Lasserre, 2004). Furthermore, the explicit attention to the joint and additive values being produced following afforestation is a novel contribution to the real option analysis in resource economics. The extension to multiple services is highly relevant, but it comes at the expense of the mathematical tractability of analysing the decision problem. We had to resort, as is often the case in such problems, to numerical solution procedures to solve explicitly defined examples.

2. State of the art

The first real-option related studies within natural resource economics were the seminal papers by Henry (1974) and Arrow and Fisher (1974), which showed in simple two period models, that with uncertain returns to irreversible development, preservation would be optimal over a larger range of expected values of development. A different strand of work on the valuation of forest resources and exercise timing under uncertainty, which is essentially also real option problems, was developed in e.g. Brazee and Mendelsohn (1988), Morck et al. (1989), Clarke and Reed (1989), Thomson (1992), and Plantinga (1998). Over time, the applications were extended to include more and more problems, such as uneven-aged management (Haigt, 1990), differentiated timber prices (Forbosh et al., 1996) and progressive income taxes (Thorsen, 1999). In all of these studies, the focus was on exercising the right to harvest a forest stand in return for the stochastically varying value of timber. Other studies assessed the optimal choice of a forest owner to enter an irreversible carbon and biodiversity protection scheme (Chesney et al., 2017).

In the current study, the decision concerned the option to forego the value of agricultural production in return receiving the benefits of timber and recreational hunting. The framework we applied here was also different and drew upon the real option literature, where McDonald and Siegel (1986) explored the value of waiting to invest when the value of investing was uncertain. This literature was first gathered in a comprehensive text book by Dixit and Pindyck (1994). This book and several applications after it, e.g. Willlassen (1998), Thorsen (1999), Ahlbrudt and Strange (1999), Saphores and Shogren (2005), Conrad and Kotani (2005), Wirl (2006) have focused on real option problems, where essentially a single option was valued and analysed. The valuation of two exclusive land-use options were first analysed by Geltner et al. (1996) and Thorsen and Malchow-Moller (2003). Malchow-Moller et al. (2004) extended this analysis to include adjacency constraints and showed that costs of adjacency constraints tend to increase with uncertainty. Jacobsen (2007) extended the problem to a situation of nested options where the classic optimal stopping problem depended on the development of two future mutually exclusive options. Jacobsen and Thorsen (2003) considered mixed species stands, where species profitability is stochastic, climate dependent and species compete for growth space. Allowing for thinning, this was a complex real option on an adjustable production mix. Kassar and Lasserre (2004) analysed the value of biodiversity protection, when multiple species were included and they were substitutable. They showed that species substitutability contained a positive value due to the real option, and that a positive correlation reduced this value. An applied nature preservation study of mutually exclusive options was presented in Blozynski et al. (2000) studying the preservation of wetlands. Morgan et al. (2008) adapted the real options approach to evaluate the optimal stopping of timber harvesting when a woodland caribou (Rangifer tarandus caribou) population became threatened with extinction. Leroux et al. (2009) modelled the optimal biodiversity extinction debt applying a novel feedback between conversion decisions and the stochastic nature of conservation benefits. Shah and Ando (2016) evaluated the opportunity costs to a private landowner of forgoing conversion use of land, either on a temporary or permanent basis, when returns from both conversion and conversion use were stochastic. Similar Frey et al. (2013) applied a real options analysis to examine the impact of flexibility in decision making under agriculture,
forestry, and agroforestry. While these studies addressed real option cases with more than one valuable asset under consideration, the assets were more or less mutually exclusive. We advance the literature on land use and real options theory by considering the case where the irreversible land use choice may lead to the provision of jointly produced stochastic assets. Such an extension of the real option approach is a novel contribution to the literature.

3. The model

We developed a general model for the case where a decision maker holds the option to put down a known fixed one-off payment and in turn receive the sum of two present value processes. In the afforestation context, this has relevance. Consider for example the landowner, who faces the problem of continuing with the present agricultural system, assuming it is generating a constant return of $F$ per unit of time to society or has the option to invest $I$ in an afforestation project. The project will generate benefits from timber and wood production as well as recreational hunting. We think of this payment as including both the planting costs and the certain present value of any tending costs including pre-commercial thinnings. It is assumed constant and in real terms. The newly established afforestation area is succeeded by an infinite series of rotations with identical provision of ecosystem services, and thus the $I$ is to be compared against the expected present value of these services.

It is assumed, that the expected present value of respectively timber and recreational hunting is observed at each point in time with values $B$ and $G$, respectively, and it is assumed that they both evolve stochastically according to a geometric Brownian motion:

$$dB = \mu_B Bdt + \sigma_B BdW$$
$$dG = \mu_G Gdt + \sigma_G GdW \tag{1}$$

Here $\mu_B$ and $\mu_G$ are constant drift terms, $\sigma_B$ and $\sigma_G$ are constant scale terms scaling the impact of the shocks arriving from $\omega$ and $\nu$, that are assumed to be increments in standard Wiener processes, possibly correlated, with $\rho$ being the correlation measure. Hence, at each point in time a decision maker can decide whether to convert agricultural land into afforestation areas, receiving the expected present value $B + G$ or continue with agriculture and receive $F$ per unit time, while retaining the option to convert at a later time step. Described by the Bellman equation this decision problem can be formulated as:

$$V(B, G) = \max \{B + G - I, Fdt + (1 + \delta)dt \cdot |E[V(B + dB + G + dG)]\} \tag{2}$$

where $\delta$ is the appropriate risk-adjusted discount rate. At each point in the continuation region, the value function must satisfy:

$$V(B, G) = Fdt + (1 + \delta)dt^{-1} [V + E(dV)] \Rightarrow \delta V = F + \delta Fdt + \frac{1}{dt} E(dV) \tag{3}$$

Using Ito’s lemma and letting $dt \to 0$, we get:

$$\delta V = F + \mu_B BV_B + \mu_G VG_B + \frac{1}{2} \sigma_B^2 B^2 V_{BB} + \frac{1}{2} \sigma_G^2 G^2 V_{GG} + \rho \sigma_B \sigma_G B G V_{BG} \tag{4}$$

where $V_B$ is the first derivative of $V(B, G)$ with respect to $B$, $V_G$ is the first derivative of $V(B, G)$ with respect to $G$, $V_{BB}$, $V_{GG}$ and $V_{BG}$ are the respective second order derivatives. $\rho$ is the possible correlation between $B$ and $G$. Inserting the Bellman equation we get:

$$V = \max \begin{align*}
  &B + G - I, \\
  &Fdt + (1 + \delta) \cdot [V + F + \mu_B BV_B + \mu_G VG_B + \frac{1}{2} \sigma_B^2 B^2 V_{BB} + \\
  &\frac{1}{2} \sigma_G^2 G^2 V_{GG} + \rho \sigma_B \sigma_G B G V_{BG}]
\end{align*} \tag{5}$$

Following the usual conditions (Dixit and Pindyck, 1994) for identifying the stopping boundary we have the value matching condition $V(B', G') = B' + G' - I$ and the smooth-pasting conditions $V_B(B', G') = 1$ and $V_G(B', G') = 1$.

Furthermore, as $B$ and $G$ evolve according to geometric Brownian motions, zero is an absorbing state for them. Thus, at these boundaries the problem reduces to a single option problem for $B$ and $G$ respectively. The solution to these problems is standard and was described in e.g. Dixit and Pindyck (1994), and briefly stated in Appendix A. The intuition of the expected value of postponing the decision to invest in afforestation is provided in Appendix B.

Because correlation may be less than perfect, the stopping boundary cannot be determined simply using the sum of the two processes as a new single process. The value of stopping will depend not only on the sum of the values, but on their relative mix as the immediate variance varies according to this. Thus, in the state space where both processes have a positive value, the problem cannot be solved analytically Curran (1994). We resorted to numerical solution procedures (Malchow-Møller et al., 2004; Thorsen and Malchow-Møller, 2003).

4. Numerical solution procedure

Based on Malchow-Møller et al. (2004) we assumed that the value function was finite as long as $\delta > \max \{\mu_B + \mu_G\}$ (McDonald and Siegel, 1986) and the problem could be solved using a value-function iteration procedure (Judd, 1998). To increase stability and iteration speed, the problem was log-normalised, which implied the variance became independent of the state, thereby standardising the calculation of the joint probability distribution. When presenting the results we converted the estimated value function and states back into the original state space values. As a base case, this study applied values of timber and recreational hunting standard deviations ranging from 0.02 to 0.10 spanning the most part of the uncertainty levels estimated in the empirical case presented below. The costs of afforestation included the value, $F$, which the decision maker forgoes if the area is afforested, and the investment cost, $I$, of afforesting the area. To simplify, we modelled these simply as a pulse cost of $I = 20$. We set the discount rate $\delta$ at 0.05. For modelling purpose we assumed that the returns from agriculture are constant, and the cost of investment (stopping) $F/I + 1 = 20$. In the numerical simulation we set $F$ to zero and $I$ to 20. If positive, the value contribution would be constant, $F/\delta$, in any case. But note that this is mathematically equivalent to e.g. $F = 1$ and $I = 0$, or $F = 0.5$ and $I = 10$ and so on.

5. Empirical example of two additive ecosystem services: timber production and recreational hunting

Real options of joint production is a generic topic, and the output of our analyses covered a realistic range of possible relative states of values $B$ and $G$ and costs $I$. As such they are illustrative of the generic aspects of the decision problem. However, for further illustrative purposes, we also evaluated the question using the case of afforestation of agricultural land with jointly produced ecosystem services. We used time series data from Denmark on timber and recreational hunting to derive parameter values for the model. Since the late 1980s, a range of European Union and Member State policies have been designed to increase the area of woodland across Europe. The Danish Parliament approved a national afforestation programme in 1989 and since 1991 it has been possible for farmers to apply for afforestation grants within this programme (Danish Forest and Nature Agency, 2010; Madsen, 2002). The afforestation programme is not restricted to native tree species, but can also cover exotics. Today the main policy objectives in Denmark pursued with the establishments of afforestation areas are to secure clean groundwater, provide recreational opportunities and long-term improved conditions for biodiversity. If an agricultural area is first set aside for forestry, it cannot return to agriculture. A farmer is paid a subsidy, and a note is made in the Danish property register on this land use entitlement, which is legally binding also for future owners. It is
very costly, if possible at all, to reverse the decision. This system makes the decision to accept an afforestation contract a legally irreversible decision. Even if legislation can be changed (i.e. if we take on a social planner perspective), the marginal decision of a single afforestation area’s change in status is legally irreversible as the alternative would not be politically feasible.

The lack of time series data on afforestation costs, and stumpage prices and costs on assortments distributions from thinnings and final harvest prevented us from estimating a times series of the net present value of timber production. Instead, we approximated the stochastic drift and volatility of the expected present value of timber production, \( B \), using historical (1911–2010) prices on beech logs (\( Fagus sylvatica \)). The time series were volume weighted gross nominal prices of large dimensioned assortments of beech logs. The data was collected from annual accounts of the Danish state forest districts between 1911 and 1996. Time series between 1997 and 2010 was based on data collected by the Danish Forest Owner Association and annual reports from a number of private forest enterprises. The parameters \( \mu_B \) and \( \sigma_B \) were estimated using the 1912–2010 time series, of the annual real CPI corrected sales price of beech logs (Statistics Denmark, 2018). The value of recreational hunting, \( G \), was based on the annual rent value from hunting reported by the Danish Forest Owners Association. The parameters \( \mu_G \) and \( \sigma_G \) were estimated using the only available CPI corrected time series, 1998–2011, of the annual average hunting rental revenues on private forest land (Danish Forest Owners Association, 2012). The regression procedures of Conrad (1997) were followed and the trend and standard deviation parameters were estimated. The time series of the recreational hunting data was too short to make the Dickie–Fuller test useful. In the case of the much longer timber price series we found that we cannot reject the null-hypothesis \( (F = 3.93) \), justifying the assumption that timber prices could be assumed to follow a Brownian motion. For both time series, we could not reject the null-hypothesis of the mean drift being equal to zero. Therefore, we assumed there were no significant trends in the data on timber values and recreational hunting \( \mu_B = \mu_G = 0 \). This assumption may seem rather conservative from society’s perspective considering current pressure on natural resources (see the introduction).

The standard deviations of the value of timber production and recreational hunting were estimated as \( \sigma_B = 0.10 \) and \( \sigma_G = 0.06 \), respectively. The values of recreational hunting and timber production was significantly negatively correlated \( (\rho = 0.864, p < 0.0001) \). However, it should be noted that the estimation of the correlation coefficient was based on the period 1998–2010. This was quite a short period and the negative correlation was mainly caused by a sudden fall in the beech timber values around year 2000. In the current study, we estimated the value function of the joint production of timber and recreational hunting using a set of different combinations of the standard deviations of B and G varying from 0.02 to 0.10, which was a range also including the empirical case.

6. Results

We started by analysing the basic properties of the decision problem and the implications of changing parameter settings on the stopping boundaries. Subsequently we estimated the optimal stopping values of the empirical case.

6.1. Stopping boundaries of additive real options

Note we have reversed the decision problem found in many real option studies, i.e. the irreversible decision is to develop an asset, which is currently preserved (Conrad, 1997; Kassar and Lasserre, 2004; Reed, 1993). In the current study, in each time step the decision maker had the opportunity of stopping the current agricultural production, investing \( I \) in afforestation, foregoing \( F \) per time unit, and gaining the expected values of timber production, \( B \), and recreational hunting, \( G \). For simplicity, we assumed that the deterministic value of \( F \) is zero.¹

Table 1 compares the value functions of a deterministic case and three different levels of \( \sigma_B \) and \( \sigma_G \) at three different states \((B, G)\). The value function of the deterministic problem, where \( B \) and \( G \) are constants since the drifts are zero, equals almost zero in the state: \( B = G = 9.974 \), since \( B + G = 19.948 < I \). The chosen step size (of 0.02) in the log-normalised numerical solution procedure resulted in the non-logged states \( B = G = 9.974 \) being those closest to value of 10. In the deterministic case, the stopping boundary is presented by all combinations of \( B + G = I \). By comparing the value function of the deterministic case with the value functions of the three stochastic problems, we estimated the well-known effects (see Dixit and Pindyck, 1994), that treating the problem in a real-options framework instead of a deterministic framework increases the value at any state. Let us present with an example. The states \( B = 9.974 \) and \( G = 9.974 \) are as mentioned above representing a combination of \( B \) and \( I \) where the value function is zero for the deterministic case, as \( B + G < I \). Assuming uncertainty on \( B \) and \( G \), \( \sigma_B = \sigma_G = 0.02 \), the value function equals 0.31. The stopping value is estimated as \( B + G - I = -0.052 \). The value of continuing farming and postponing the decision to afforest is estimated as \( V(B, G) \) minus the stopping value. It is estimated at 0.24. Since the value of continuation is positive it is still not optimal to afforest when \( B = G = 9.974 \). It demonstrates that the value function increases with increasing uncertainty meaning with increasing uncertainty on \( B \) and \( G \) the value of continuing postponing the decision to afforest increases. The value of continuation is estimated at 1.35 when \( \sigma_B = \sigma_G = 0.08 \), which is estimated 0.24 when \( \sigma_B = \sigma_G = 0.02 \). In the deterministic case the value function is estimated as \( B + G - I \), which for the two states, \( B = G = 10.176 \) and \( B = G = 10.381 \) result in 0.352 \((10.176 + 10.176 - 20)\) and 0.762 \((10.381 + 10.381 - 0)\) (see second column Table 1). For these two states, it is optimal to afforest because the value function equals the stopping value. However, introducing uncertainty on the same two states of \( B \) and \( G \) implies that their value functions are larger than their stopping value, i.e. for \( V(11.246, 11.246) \) 1.655 > 0.762, and it is optimal to postpone the decision to afforest. For all three states and two uncertainty levels presented in Table 1 it is noted that we are still in the continuation region, where afforestation is not optimal under uncertainty.

We further analysed the base case by setting the correlation between \( B \) and \( G \) to zero and estimated the stopping and continuation regions for a range of standard deviations. Fig. 1 shows the shape of the continuation region (south-west of each plot), i.e. the value combinations of \( B \) and \( G \) at which it is optimal to continue agricultural production. Fig. 1 also shows the stopping regions (north-east of each plot), i.e. the values of \( B \) and \( G \) where it is optimal to afforest the land to gain timber and recreational hunting values, while foregoing agricultural income. The deterministic stopping boundary (where \( B + G = I \) ) is also included in Fig. 1. The regions are shown for four different symmetric standard deviations, when \( \sigma_B = \sigma_G = (0.02, 0.04, 0.06, 0.08) \) and assuming correlation between timber and recreational hunting services was zero. Increasing uncertainty on \( B \) and \( G \) we expected the optimal stopping boundary to shift outwards. It would require higher threshold values of \( B \) and \( G \) to exercise the option of afforestation. This is indeed what is shown in Fig. 1.

Fig. 1 also demonstrates that departing from the single option case to more jointly produced environmental services will increase the value of options. As an example, we observe for the single option cases, at states \( (0, 12) \) and \( (12, 0) \) respectively, when \( \sigma_B = \sigma_G = 0.02 (\rho = 0) \), that we are still in the continue region of the single option problem. However, if the options are jointly produced we would horizontally and vertically be located at the state \( (12, 12) \), which is inside the stopping region of the joint options problem. This is a simple and expected result

1Relaxing this assumption causes only a parallel shift of the stopping boundary, as any \( F > 0 \) corresponds to adding \( F/I \) to the pulse cost \( I \).
of the additivity of the jointly produced ecosystem services. Changing the size of standard deviations and the correlation affects the magnitude, but not the qualitative implication of this result of joint options.

The stopping boundaries appear to be linear, at least within the range of $B$ and $G$ shown here. However, estimating the Euclidean distance between the stopping boundaries and the line between the extreme points of the boundary we did find a minor convexity. Since this may also be a result of the numerical estimation procedure, we did not conclude on this. As is seen Table 2, maintaining symmetric standard deviations of 0.08 but increasing the correlation between $B$ and $G$ at +0.5 the stopping boundary increases. It reflects that positive correlation increases uncertainty and in turn the value of continuation and the stopping values required for changing from agriculture to forestry and the production of timber and recreational hunting benefits. The opposite is the case when setting the correlation at −0.5. This means that in this case, less than perfect (positive) correlation reduces the value of the option on a sum of stochastic assets, here in the form of timber and recreational hunting services, and the more the lower the correlation.

In the previous examples, we assumed symmetric standard deviations. Fig. 2 illustrates the impacts of decreasing uncertainty on $B$, maintaining uncertainty on $G$. The slope of the stopping boundary becomes more negative, as expected, since lower threshold values of $B$ are required to exercise the option of getting $B$ and $G$. Otherwise the pattern is similar.

### 6.2. Optimal stopping boundaries of the empirical case

The optimal stopping boundary of the empirical case of timber production and recreational hunting is shown in Fig. 3. Since the standard deviation of recreational hunting is smaller than the standard deviation of timber production, higher values of timber production than hunting are required before stopping agricultural production. The empirical data pointed at an almost perfect negative correlation ‘pushing’ the stopping boundary closer to origo than if the correlations have been zero or a positive value of say 0.5. As mentioned above, the estimation of correlation may be sensitive to the length of time period sampled, which in this case was quite short (1998–2010).

### 7. Concluding discussion

The current study addresses the importance of including investment theory and in particular, the implications of uncertainty of the future value of the ecosystem services when the services may rather be jointly produced than mutually exclusive. In addition, the modelling should acknowledge that a decision maker may hesitate to invest in forest and nature restoration or conservation since such decisions may be irreversible (one example is the European Natura 2000 network of set a

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Value functions $V(B,G)$ at three different states for deterministic cases and for three levels of standard deviation ($\rho = 0$). Note the relatively small sizes of the value function as cost is included.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V(9,974, 9,974)$</td>
<td>$\sigma_B = \sigma_G = 0.02$</td>
</tr>
<tr>
<td>$-0.05$</td>
<td>$0.308$</td>
</tr>
<tr>
<td>$0.352$</td>
<td>$0.485$</td>
</tr>
<tr>
<td>$0.762$</td>
<td>$0.763$</td>
</tr>
</tbody>
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<table>
<thead>
<tr>
<th>Table 2</th>
<th>Value function, $V(B,G)$, at three different correlations ($\sigma_B = \sigma_G = 0.08$).</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V(7,243, 7,243)$</td>
<td>$\rho = +0.5$</td>
</tr>
<tr>
<td>$0.306$</td>
<td>$0.19$</td>
</tr>
<tr>
<td>$1.746$</td>
<td>$1.468$</td>
</tr>
<tr>
<td>$2.895$</td>
<td>$2.671$</td>
</tr>
</tbody>
</table>
side areas, where conventional management is no longer permitted) and the expected future values of the environmental services are uncertain. Further, the global spending on nature restoration or preservation is, despite being inadequate, considerable. Waldron et al. (2013) estimated the total annual expenditures on global biodiversity in 2005 US dollars at approximately $21.5 billion. We have not found an estimate of the global investments in nature restoration. The European Commission has estimated that the annual cost of financing the European protected area network, Natura 2000, is EURO 6 billion per year (European Commission, 2019). If all factors are taken into account properly, economic benefits generated by forest ecosystem services (such as food and fiber, water regulation, carbon sequestration, air- and water-purification, recreational benefits, and aesthetic and spiritual values) may be outsizing the costs of investing in ecosystem restoration and conservation. Costanza et al. (2014) repeated their 1997 study (Costanza et al., 1997) of the value of the world’s ecosystem services with updated data and estimated the loss of eco-services from 1997 to 2011 due to land use change and ecosystem degradation at USD 4.3–20.2 trillion per year. Despite a critique of additionality their studies indicate the importance of such values. Ecosystem services are often bundled and uncertain. Real options theory may be appropriate to address the nature of such decision problems (Clarke and Reed, 1989; Conrad, 1997). The current study aimed at extending the real-options theory to reflect this rather complicated stochastic control problem of

![Fig. 2. Changes in stopping boundary when standard deviations are asymmetric. $\sigma_B = 0.08$ and $\sigma_G$ are presented for three levels of standard deviation, $\sigma_G = 0.08, 0.05,$ and $0.02$. $\rho = 0.0$.](image)

![Fig. 3. Stopping and continuation region for the empirical example of timber production (B) and recreational hunting (G). The base case is $\sigma_B = 0.10$, $\sigma_G = 0.06$, and $\rho = −0.864$. For comparison $\rho = 0.0$ and $+0.5$ are included.](image)
joint real options, using timber and recreational hunting as an example of two jointly provided services. Many other decision problems are recognized jointly produced ecosystem services, with carbon storage and biodiversity being some examples (Phelps et al., 2012).

The current study extends the real option literature by considering an application of jointly produced real options. Treating jointly produced options as complementary, we identified stopping rules that differed from those of earlier studies. The joint production increased the value function and the incentives to convert agricultural land to multiple-use forests. The results demonstrated that increasing uncertainty and positive correlation between the future values of timber production and recreational hunting decreased the incentive to afforest. The study also extends Jacobsen and Helles (2006), who numerically solved the joint production of various timber assortments, by studying the implications of various correlations on the stopping boundary. It is also found that as the value of one of the two options becomes relative small compared to the other option, the stopping value approaches the one of a single option.

One main result of the analysis demonstrated that reducing the correlation between options reduced the value of the option. This is contrary to the well-known results from portfolio theory and risk diversification, where less than perfect correlation would enhance the value of a set of assets. The effect arises because decreasing correlation decreases the variance of the combined jointly held options yield. In portfolio theory, such decreased variance has a value, because the value is performed under the assumption of risk averse. In our case, however, we valued a portfolio of assets – real investment options – under risk neutrality, and found a value decrease. Thus, this finding is entirely due to a reduced value of waiting brought about by the reduced variance. We note that the discount rate can be interpreted as the market-based risk adjusted discount rate, and thus accounting for the market price of risk. According to the capital asset pricing model, the risk aversion of the individual forest owner does not affect his valuation of the afforestation asset, but it does affect the proportion of risky assets relative to risk-free assets, the owner will hold. Analysing this is outside the scope of our analysis.

The finding that reduced correlation reduces the value of the option is contrary to earlier results on the value of a real option on a set of assets, e.g. Geltner et al. (1996) Thorsen and Malchow-Moller (2003) and Malchow-Moller et al. (2004). The reason for this is that in these earlier studies, the option concerned a set of exclusive options, whereas the current study addresses a set of joint options. Exclusivity implies that decreasing correlation raises the chance of taking advantage of high outcomes (of one of the options), and hence the value of waiting. This finding may be important when addressing the value of bundles of several options. It is well-known that forests provide a range of ecosystem services, e.g. erosion control, improved quality of drinking water, climate mitigation and carbon storage, biodiversity, aesthetics (e.g., Filyushkina et al., 2016). This may increase the societal need for restoring forest ecosystems on agricultural land. This study has showed, that if expected values of several ecosystem services included in the bundles are positively correlated, it would increase uncertainty and the stopping values required for changing from agriculture to forestry. On the other hand, the more un-correlated they are, the more it reduces the uncertainty and stopping values required to convert the agricultural field into forests providing a multiple of ecosystem services. In the current study, we analysed a case of joint production of real options in forestry. Investment in joint production, where several outputs are result from a single productive activity, is a general case in several sectors. An example is the refining of crude oil, in which gasoline, kerosene, light heating oil and other mineral oil products are produced. Another example is the production of several products e.g., butter, and butter milk, skimmed milk and cream from milk. We suggest that the method applied in the current study could be relevant for investigating the value of jointly produced real options in other fields and sectors.

That decreasing correlation of joint production did not increase the option value has important implications for management as it means that in these situations afforestation is more likely to be favourable. At the same time, we assumed that when the option was exercised, we received the expected present value of timber and recreational hunting. Obviously, this value would remain uncertain also after the irreversible decision of afforestation has been implemented. Once afforested, decisions can no longer be reversed and in this phase negative correlation between the timber and hunting values could have a risk reducing value for the risk-averse owner. This would, all other things being equal, tend to increase stopping value further for the risk-averse owner relative to the risk-neutral owner we have modelled. This type of study could help policy makers and landowners understanding how they should ideally make decisions (normative approach) in a given context or, perhaps rather, help us understand what may actually drive decisions in a context of additive options (positivist approach). Afforestation is an irreversible decision in the sense that planting trees on agricultural fields in Denmark is legally binding if supported by a public subsidy. Even if not binding, the costs of removing trees, stumps and soil preparation are considerable. Agricultural crop production is usually in one year cycles and the landowner can change crops and adapt to new circumstances much quicker than in forestry. Therefore landowners lose flexibility and this may trigger their preference to stay in agriculture and postpone the decision to convert to forest, a result also pointed out by a number of real option studies (Malchow-Moller et al., 2004; Malchow-Moller and Thorsen, 2006; Thorsen, 1999; Thorsen and Malchow-Moller, 2003). Such impact on landowner decisions was clearly documented by Yemshanov et al. (2015), who applied specific price and cost factors to model real option values of afforestation in Alberta, Canada and compared with land use conversion data.

We found that the joint production increased the value function and the incentives to convert agricultural land to multiple-use forests. However, uncertainty related to economic consequences of catastrophic events such as windthrows (Meilby et al., 2001), fire (Reed, 1984), and epidemic pests (Reed and Errico, 1987) and diseases may reduce the incentives of land owners to invest in afforestation. Including such aspects will add to the realism of the decision problem a land owner is facing, and should be considered in future studies.

We assumed that the value of afforestation is a joint product of timber production and recreational hunting. The later was estimated applying time series data on revenues from hunting leases at forest estates. We disregarded the recreational value of agricultural land in a traditional crop production system, which may also include some hunting values. A study by Lundhede et al. (2015) analysed the market for hunting leases in Denmark using the hedonic method on data obtained from Danish hunters. They found that the hunting value of forest areas is considerably higher than for agricultural fields. Although the value of hunting on agricultural fields is much lower than on forest land, including the value of recreational hunting on agricultural fields may reduce the incentive to afforest.

This study applied a discount rate of 5%. Though it may appear in the high end, it is not unusual to find forest economic studies applying discount rates between 2 and 12% (Brukas et al., 2001). However, a discount rate of 5% may still be relevant for alternative land use investments to agriculture. The choice of discount rate would not change the conclusions made in the current study. If we assumed a lower discount rate than 5% the value function increases and the stopping boundaries in Fig. 1 would move towards north-east. Similar, the stopping boundaries would move towards south-west if the discount rate is higher than 5%.

We demonstrated the real option value of joint production of only two ecosystem services. If we included even more services, which were jointly produced with G and B, e.g. carbon storage and drinking water, and if they contribute positively to the value function, the direct effect would be to increase the value of stopping, and hence afforestation would be relevant for even more states of nature. However, the effects also depend on the level of uncertainty associated with the additional
joint production outcomes. In case the additional services are positively correlated with other services, the overall result is a higher level of uncertainty and this in turn reduces the value of stopping, and reduces the set of states of nature where afforestation is optimal. Thus, the aggregate effect on the set of state where afforestation is optimal depends on these opposite forces.

In this study, we assumed ecosystem services are produced jointly with no trade-off between them. In this case we found it reasonable to assume that afforesting land may benefit timber production and recreational hunting. However, the value of recreational hunting, as an example, may be spatially disconnected since the place where the service is produced (costs are born) may not be the same, or at least the only, place where the value is harvested (neighbouring hunting areas may benefit from the afforestation project). Similar, specific taxonomic groups of biodiversity with large home ranges or migration patterns, e.g. mammals, birds and insects may also depend on structures and habitat availability at a landscape level. Other services such as carbon storage in above and below ground biomass may be much more spatially connected to the primary production of specific areas in question. Therefore, one major challenge of the valuation of ecosystem services is to understand the dynamics and links of ecosystem services facilitating quantification and determination of trade-offs between ecosystem services (Carpenter et al., 2009, 2006). In the current study, correlation was treated as an aggregate measure of both correlation between the technological production of environmental services and their economic values. In real world decision problems, these may need to be estimated and treated separately. Studies have directed their attention towards the need for developing a theoretical and methodological understanding of the multiple and non-linear relationships among ecosystem services (Bennett et al., 2009). Although studies have attempted to quantify the relationship (Kremen and Ostfeld, 2005) and mapped the supply and demand of services (van Jaarsveld et al., 2005) there is a need for estimating the spatial correlation in production and economic terms. This study demonstrated that the implications for decision making may be considerable.

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Appendix A

As B and G evolve according to geometric Brownian motions, zero is an absorbing state for them. Thus, at these boundaries the problem reduces to a single option problem for B and G respectively. The solution to these problems is standard and is described in e.g. Dixit and Pindyck (1994), and is repeated here. The values of the single options at the boundaries are respectively:

\[
V(B, 0) = \frac{V(B) + F}{\beta}
\]

where

\[
A_B = \frac{G - \frac{F}{\beta} - I}{G^*\beta} = \left(\frac{G_B}{G^*} - \frac{F}{\beta} + \frac{I}{\beta}\right)
\]

and

\[
B_B = \frac{B - \frac{F}{\beta} - I}{B^*\beta} = \left(\frac{B_B}{B^*} - \frac{F}{\beta} + \frac{I}{\beta}\right)
\]

The optimal stopping boundaries are given by

\[
G^* = \frac{G_B}{G_B - I}
\]

\[
B^* = \frac{B_B}{B_B - I}
\]

Appendix B

The following presents a simple example of the optimal stopping problem and the intuition of the value of waiting. The decision problem is to continue with open non-productive field or make the irreversible choice of afforesting the land and start producing timber values (B) and recreational hunting (G). The opportunity cost of afforesting that the decision maker faces is given by I, which represents the cost of afforesting the area.

We assume that current (period 1) values of timber and recreational hunting are 1000, respectively, but the values can change with probability ½ to the permanent values of 500 or 1500 in the next period for each of them. If the farmer chooses to undertake the investment in period 1, the farmer will have the following expected net present value if the interest rate r, is 0.05:

\[
\text{Expected Net Present Value} = 1000 \times (1 + 0.05)^{-1} + 1500 \times (1 + 0.05)^{-2} + 500 \times (1 + 0.05)^{-1}.
\]
\[-I + 2000 + \sum_{i=1}^{\infty} \left( \frac{1}{1.05} \right)^i \left( \frac{1}{4} (1,500 + 1,500) + \frac{1}{4} (500 + 500) + \frac{1}{2} (1,500 + 500) \right) = 42,000 - I\]

It is profitable for the farmer only to invest in period 1 if \(I < 42,000\).

If the farmer postpones the decision until period 2, the farmer can observe whether the values of \(B\) and \(G\) will move up or down. In case it moves up, the period 2 present value of investing in afforestation is \(I = 63,000 - I\), and if both \(B\) and \(G\) move down the value is \(\frac{1}{105} (500 + 500) - I = 21,000 - I\). Thus, if \(I > 63,000\) it will never be optimal to invest in afforestation, and nothing is gained or lost by postponing the decision. Similarly if \(I < 21,000\), it will always be optimal to invest and it should optimally be done in period 1.

If, on the other hand, \(21,000 < I < 63,000\) the farmer may choose to invest in period 2 if and only if the value of at least one of the joint options has moved up. In case of both \(B\) and \(G\) values move up in period 2, which happens with probability \(P = \frac{1}{4}\), the period 1 net present value of the investment decision is given by:

\[-I \times \frac{1}{1.05} + \sum_{i=1}^{\infty} \left( \frac{1}{1.05} \right)^i \left( \frac{1}{4} (1,500 + 1,500) \right) = 15,000 - \frac{I}{4.2}\]

In case \(B\) moves up and \(G\) down in period 2, and vice versa, which happens with probability \(P = \frac{1}{2}\), the period 1 net present value of the investment decision is given by:

\[-I \times \frac{1}{1.05} + \sum_{i=1}^{\infty} \left( \frac{1}{1.05} \right)^i \left( \frac{1}{2} (1,500 + 500) \right) = 20,000 - \frac{I}{2.1}\]

Thus, postponing the decision to invest and only undertaking it if either \(B\) or \(G\) or both move up, has the period 1 net present value:

\[15,000 - \frac{I}{4.2} + 20,000 - \frac{I}{2.1} = 35,000 - \frac{3I}{4.2}\]

This should be compared with investing in period 1. If:

\[35,000 - \frac{3I}{4.2} > 42,000 - I \Leftrightarrow I > 24,500\]

it is optimal to postpone the decision. Note that in the deterministic case, investment would have taken place for any \(I < 42,000\), whereas with the value of waiting, \(I\) has to be < 24,500 for stopping and investment in period 1 to be optimal.

Thus, in summary:

- \(I > 63,000\) then it will never be optimal to invest in period 1 nor at any later time period,
- \(42,000 < I < 63,000\) then postpone the decision and invest in period 2 only if both \(B\) and \(G\) moves up,
- \(24,500 < I < 42,000\) then postpone the decision and invest in period 2 if either \(B\) or \(G\) or both moves up,
- \(I < 24,500\) then always invest in period 1.

Now we illustrate the effect of correlation (implicitly assumed zero above). Assume that:

\[Pr(X_u = 1,500) = Pr(X_d = 500) = Pr(X_u = 1,500) = Pr(X_d = 500) = \frac{1}{2}\]

where \(u\) and \(d\) denote when prices on \(B\) or \(G\) go up and down, respectively.

Assuming the following symmetry requirements:

\[Pr(X_u, X_u = 1,500) = Pr(X_u, X_d = 500) = \frac{\rho + 1}{4}\]

\[Pr(X_d, X_u = 1,500) = Pr(X_d, X_d = 500) = \frac{1 - \rho}{4}\]

If we assume, that the investment will be undertaken in period 2 if at least one of \(B\) and \(G\) move up then the period 1 net present value of postponing the decision to period 2 can be computed as:

\[\frac{\rho + 1}{4} \left( \sum_{i=1}^{\infty} \left( \frac{1}{1.05} \right)^i 3,000 - \frac{I}{1.05} \right) + 2 \frac{1 - \rho}{4} \left( \sum_{i=1}^{\infty} \left( \frac{1}{1.05} \right)^i 2,000 - \frac{I}{1.05} \right) =
\]

\[35,000 - 5,000\rho - \frac{3I}{4.2} + \frac{\rho I}{4.2}\]

This value should be compared to the value of undertaking the investment in period 1, which is 42,000 – I. It is optimal to postpone the decision as long as:

\[I > \frac{21,000\rho + 29,400}{1.2 + \rho}\]

This is illustrated in Fig. B1, and thus we see that the stopping rule is sensitive to correlation in a highly non-linear fashion.


